

Chapter - 9**Average**

9. **Average:** It is also called **mean**.

Please note that here only arithmetic mean, called as average, is our concern. No standard deviations or statistics.

9.1 **Average of two numbers.**

Let A be the average

$$A = \frac{\text{Sum of the 2 numbers}}{2}$$

9.1.1 Find A of (1,5); (2,6); (8,10) etc

Ans: Mean of (1,5) = $\frac{1+5}{2} = 3$

Mean of (2,6) = $\frac{2+6}{2} = 4$

Mean of (8, 10) = $\frac{8+10}{2} = 9$

9.1.1 Examples:

- Sometimes Mr. G eats 10 idleys for breakfast. Sometimes he eats only 6 idleys. What is the average? (Or how many idleys shall I make and keep ready for him)?
Ans.: 8
- Last week tomato was Rs. 12/kg. This week it is Rs. 8/kg. What is the average price of tomato (in April in Mysore)? Ans: Rs. 10
- Some good motorbikes cost _____, Some other motorbikes cost _____. Approximately how much money is needed for a medium level motorbike? (Use suitable numbers here).
- Generate information from students asking for mean of 2 items.

Exercise:

- Maximum temperature (some day in May 2009, at Mysore) was 38°C. Minimum was 26°C. What was the average on that day?
- Yesterday's average temperature was 32°C. Day before yesterday it was 36°C. What do you expect today's average temperature?
- A Scooter gives 50 km per liter of petrol on highway. A liter of petrol is Over if the same scooter runs for 30 km inside city. What is the average mileage given by this scooter? (Mileage = distance for 1 liter of fuel).

9.1.2 When only 2 numbers are involved mean can be looked at as the midpoint (or mid value).

a. A of 8 and 10 =? $A = \frac{8+10}{2} = \frac{18}{2} = 9(OK)$

b. A of 18 & 20 =? $A = \frac{18+20}{2} = \frac{38}{2} = 19$

c. A of 78 & 80 =? $A = \frac{78+80}{2} = \frac{158}{2} = 79$

Instead try: Mid value between 8 & 10 = 19

Mid value between 18 & 20 = 19
 Mid value between 78 & 80 = 79

d. Now try 123458 and 123498

- 9.1.3 In averaging two numbers (large or small) midpoint concept is ok.
 Since average is, in any case, an approximation it is ok to round off.

Thus in 9.1.3 (d) above 123458, 123498 $A = 123478$

But you can make these as 123460, 123500 $\therefore A = 123480$

- 9.1.4 In the above try:

a. 123458, 123578. $A = ?$

b. Round off to 123460, 123580

Keep 123 out 460, 580

Difference = 120

Half of difference = 60

\therefore Value = 460 + 60 or 580 – 60 = 520

Now add 123:

$\therefore A$ of 123460, 123580 = 123520

Doing such things without calculators increases self-confidence and removes the fear of simple arithmetic.

- c. In the above example it is OK to make still more approximations:

123458, 123578 Can be 123500, 123600 $A = 123550$

- d. Compare now a, b, c

- Correct was 123518
- First approximation was 123520
- Rough value was 123550

9.1.5 Exercise

- Find the mean by finding the midpoint.
 - Distance of 3 and 5 km.
 - 1 and 2
 - Rupee equivalent of 1 US dollar: Rs. 42, Rs 48
- Find mean by approximation & mid point.
 - Population of a place 9 lakhs 9 thousand and 10 lakhs one thousand.
 - 9900 and 11100
 - 9912, 11162

9.2 Average of many numbers:

In real life situation average of many numbers are very much needed.

Rule: $A = \frac{\text{Sum of all the numbers}}{\text{Number of items}}$

$$\text{E.g.: For } 2, 4, 6, 8, 10. \quad A = \frac{2+4+6+8+10}{5} = \frac{30}{5} = 6$$

Later: Negative (-) number concept & hence assume mean value.

Keep common high numbers out and do only small numbers.

9.2.1 Marks obtained in an examination.

E.g.: 6 subjects 600 marks

Average: Add all marks; divide by 6

[If it is 625 marks, this small idea leads to percentage concept].

9.2.2 Average expenditure in a house (say on electricity) (over 3 or 10 months)

9.2.3 Average rainfall in a place (say over the last 10 years).

9.2.4 Average price of a famous company's share (say over last 4 weeks)

Extend this list using the students' ideas.

Students can take these values from a newspaper and make their own questions.

9.2.5 Exercises:

- Total marks obtained by a student is 365 total subjects. What is the average?
- If an average of 35 marks are needed for passing. What are the total marks required for a total of 6 subjects?
- In (b) above what, for I class result?
- In (c) above, which total marks will be called distinction?
- In a class of 9 students, marks obtained by them in maths are as follows: 10, 15, 15, 30, 35, 45, 50, 55, 85. What is the average in this class?
- In (e) above one more comes and gets zero marks. What is the new average?

Exercises - Chapter 9

Ex. IX.1 Ages of children in 9th standard were: 14, 15, 13, 14, 15, 16, 13, 15, 14, and 13. Find the average age?

Ex. IX.2 In (1) above two more persons joined. Their ages were 18 and 19. What is the new average?

Ex. IX.3 In a KG class children's ages were: (in years and months; y, m):
3y, 4y, 2y 10m, 2y 11m, 3y 1m, 3y 6m, 4y 2m, 4y 6m, 3y 8m, 3y. Find the average age in years and months? [1 year = 12 months]

Ex. IX.4 Marks obtained in a class are tabulated:

Marks	No. of Persons
0	2
5	3
12	15
13	10
14	5
15	2
20	2
25	1

Maximum Marks: 25
None was absent.

- How many students (total) wrote the exam?
- How many got zero marks?
- How many got full marks?
- How many first class marks?
- What is the class average?

Ex. IX.5 Mean Values and a branch of mathematics called statistics are clearly related. Do you know some words related to statistics? If yes, write down

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Chapter - 10**Fractions - 1**

- 10.1 If you have 2 items (one each for 2 of you) and 2 more join what will you do? Share by 4 so that each gets $\frac{1}{2}$.

Write it down:

a. $2 \text{ items} / 2 \text{ persons} = \frac{2}{2} = 1$

b. $2 \text{ items} / 4 \text{ persons} = \frac{2}{4} = \frac{1}{2}$

Reverse $1 + 1 = 2 \times 1 = 2$

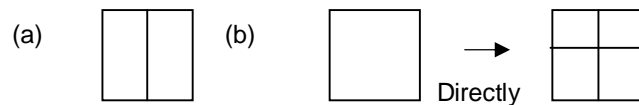
$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 4 \times \frac{1}{2} = 2$$

- 10.2 Do the same as 13.3.1, but now only with one item.

a. $1 \text{ item} / 2 \text{ persons} = \frac{1}{2}$ (half)

b. $1 \text{ item} / 4 \text{ persons} = \frac{1}{4}$ (quarter)

Take a concrete example (piece of paper). First show (a) & (b) above



(c) Now take one half piece of (a) above and make it into 2 parts, i.e. $\frac{1}{4}$

Now say $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$


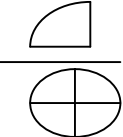
10.3 Handwork Exercise:

Take rectangular or square or circular piece of paper. Fold, cut and show:

a. $\frac{1}{2}$ b. $\frac{1}{3}$ c. $\frac{1}{4}$ d. $\frac{1}{6}$ e. $\frac{1}{24}$ f. $\frac{1}{12}$

- 10.4 In (10.3) above, create your own questions.

Eg: $\frac{*}{***} = ?$ Ans = $\frac{1}{3}$ $\frac{\square\square}{\square\square\square} = ?$ Ans = $\frac{2}{3}$

Eg:  If this is 1,  = ? Ans = $\frac{1}{4}$

- 10.5 Go to 3 items to be shared by 3. It gives one each $\frac{3}{3} = 1$

Now 3 more join. Then what happens? See 13.3.1 (a) & (b) & do similar here.

a. $3 \text{ items} / 3 \text{ persons} = \frac{3}{3} = 1$

b. 3 items / 6 persons = $\frac{3}{6} = \frac{1}{2}$ = half

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \left(\frac{1}{2} + \frac{1}{2}\right) + \left(\frac{1}{2} + \frac{1}{2}\right) + \left(\frac{1}{2} + \frac{1}{2}\right) = 1 + 1 + 1 = 3$$

Eg: $\frac{1}{2}$ added 6 times = $\frac{1}{2} \times 6 = 3$

Do:

a. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

b. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

c. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

d. $\frac{1}{5} + \dots\dots\dots$ (How Many) = 1 e. $\frac{1}{8} + \dots\dots\dots$ (How Many) = 1

f. In (e) how many = 2

10.6 Addition of simple fractions.

a. $\frac{1}{2} + \frac{1}{2} = 1$ (Use coins) i.e., 50 Paise + 50 Paise = Re 1

$$\left[\frac{1}{2} \text{ Re} + \frac{1}{2} \text{ Re} = \text{Re } 1 \right]$$

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \left(\frac{1}{2} + \frac{1}{2}\right) + \frac{1}{2} = 1 \text{ plus } \frac{1}{2} \text{ Here 3 coins of 50 Paise}$$

b. $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$ (Show graphically)

c. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \left(\frac{1}{4} + \frac{1}{4}\right) + \left(\frac{1}{4} + \frac{1}{4}\right) = \frac{1}{2} + \frac{1}{2} = 1$

d. or $\left(\frac{1}{4} + \frac{1}{4}\right) + \frac{1}{2} = 1$ or $\left(\frac{1}{4} + \frac{1}{4}\right) + \frac{2}{4} = 1$

e. $\frac{1}{3} + \frac{1}{3} = 2 \times \left(\frac{1}{3}\right) = \frac{2}{3}$

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \square \\ \hline \end{array} \text{ All pieces put together } = 1$$

10.7 Now give the rule:

a. $\frac{1}{2} + \frac{1}{2} = \frac{1+1}{2} = \frac{2}{2} = 1$

b. $\frac{1}{4} + \frac{1}{4} = \frac{1+1}{4} = \frac{2}{4} = \frac{1}{2}$

c. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{1+1+1}{4} = \frac{3}{4}$

d. $\frac{1}{4} + \frac{1}{4} + \frac{2}{4} = \frac{1+1+2}{4} = \frac{4}{4} = 1$

But we know that $\frac{2}{4} = \frac{1}{2}$

$$\therefore \frac{1}{4} + \frac{1}{4} + \frac{1}{2} = \frac{1 + 1 + 2}{4} = \frac{4}{4} = 1$$

10.8 Exercises (Addition of only 2 fractions). Denominators are the same.

E.g.1. $\frac{1}{5} + \frac{1}{5} = \frac{1 + 1}{5} = \frac{2}{5}$

E.g.2. $\frac{1}{4} + \frac{3}{4} = ?$ Ans: $\frac{1 + 3}{4} = \frac{4}{4} = 1$

Do:

a. $\frac{1}{6} + \frac{1}{6}$ b. $\frac{1}{6} + \frac{4}{5}$ c. $\frac{1}{2} + \frac{1}{2}$ d. $\frac{1}{2} + \frac{3}{2}$ e. $\frac{1}{7} + \frac{1}{7}$ f. $\frac{1}{7} + \frac{5}{7}$

g. $\frac{1}{7} + \frac{6}{7}$ h. $\frac{1}{11} + \frac{10}{11}$ i. $\frac{1}{99} + \frac{98}{99}$ j. $\frac{89}{99} + \frac{10}{99}$ k. $\frac{17}{34} + \frac{51}{34}$ l. $\frac{1}{17} + \frac{6}{17}$

m. $\frac{3}{17} + \frac{31}{17}$ n. $\frac{1}{986543} + \frac{1}{986543}$ o. $\frac{986500}{986543} + \frac{43}{986543}$

10.9 a. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{1 + 1 + 1}{3} = \frac{3}{3} = 1$ [This is not $\frac{3}{9}$]

b. $\frac{1}{3} + \frac{1}{3} = \frac{1 + 1}{3} = \frac{2}{3}$

c. $(\frac{1}{3} + \frac{1}{3}) + \frac{1}{3} = \frac{2}{3} + \frac{1}{3} = \frac{2 + 1}{3} = \frac{3}{3} = 1$

(a) is the same as (c)

d. $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{2 + 2 + 2}{3} = \frac{6}{3} = 2$ [This is not $\frac{6}{9}$]

e. $\frac{2}{3} + \frac{2}{3} = \frac{4}{3} = 1 \text{ Plus } \frac{1}{3}$

(Show this graphically or cut pieces).

Exercise:

All the above can be demonstrated by folding / cutting out of cardboard or on a graph paper. Students can do it by both the methods and show to teacher / one another.

10.10 Go to negative (subtraction).

E.g.: $1 - \frac{1}{2} = \frac{2 - 1}{2} = \frac{1}{2}$

(If this is tough, go to $\frac{1}{2} + \frac{1}{2} = 1$ take away from RHS one half). What is left will be $\frac{1}{2}$.

Exercise: Do by craft method and by graph paper

a. $1 - \frac{1}{3}$

b. $1 - \frac{2}{3}$

c. $1 - \frac{1}{6}$

d. $1 - \frac{5}{6}$

e. $1 - \frac{1}{4}$

f. $\frac{4}{3} - \frac{1}{3}$

g. $1 - \frac{1}{16}$

h. $1 - \frac{15}{16}$

i. $2 - \frac{3}{8}$

j. $1 - \frac{3}{8}$

Chapter - 11

Fractions - 2

11. Fractions: Extension of division.

Go back to division and revise.

- 11.1 Division and remainder:

- a. Select any single digit number to be divided by any other single digit number.

Make a set A (1 to 5); another set B (6 to 10)

Make $A \div B$

E.g.: $1 \div 6$ written as $1/6$ division not possible.
 $3 \div 7$ written as $3/7$ division not possible.

Whenever you say "not possible" to divide it is called a FRACTION.

- b. In (a) above do $B \div A$

E.g.: $6 \div 1$ i.e. $\begin{array}{r} 6 \\ 1 \end{array} = 6$ + no remainder
 $7 \div 2$ $\begin{array}{r} 7 \\ 2 \end{array} = 3$ + remainder 1
 $8 \div 4$ $\begin{array}{r} 8 \\ 4 \end{array} = 2$ + no remainder
 $9 \div 4$ $\begin{array}{r} 9 \\ 4 \end{array} = 2$ + remainder 1
 $9 \div 3$ $\begin{array}{r} 9 \\ 3 \end{array} = 3$ + no remainder
 $10 \div 3$ $\begin{array}{r} 10 \\ 3 \end{array} = 3$ + remainder 1
 $10 \div 5$ $\begin{array}{r} 10 \\ 5 \end{array} = 2$ + no remainder

If there is no remainder the answer is an INTEGER (i.e. it is a whole number).

If there is remainder, the answer contains FRACTIONS.

Thus $\frac{7}{2} = 3 + \frac{1}{2}$ also written as $3 \frac{1}{2}$

$\frac{9}{2} = 4 + \frac{1}{2}$ also written as $4 \frac{1}{2}$

$\frac{10}{3} = 3 + \frac{1}{3}$ also written as $3 \frac{1}{3}$

(Sometimes a number containing both integer and fraction is called **MIXED FRACTION**)

11.1.1 Exercises: Write down <1 or >1

E.g.: $\frac{1}{3}$ Ans: <1, $\frac{4}{3}$ Ans: >1, $\frac{3}{3}$ Ans: 1

a. $\frac{9}{8}$ b. $\frac{8}{8}$ c. $\frac{7}{8}$ d. $\frac{19}{20}$ e. $\frac{99}{100}$ f. $\frac{101}{100}$

g. $\frac{1234}{1235}$ h. $\frac{1238}{1235}$

11.2 Division – bigger numbers: Follow the same method as above but use the written format as shown in earlier Chapter.

11.2.1 Examples:

(i) $987 \div 8$

$$\begin{array}{r} 123 \\ 8 \overline{)987} \\ \underline{8} \\ 18 \\ \underline{16} \\ 027 \\ \underline{024} \\ 003 \end{array}$$

Ans: 123 remainder = 3

$$\therefore \frac{987}{8} = 123 \frac{3}{8}$$

(ii) $987 \div 12$

$$\begin{array}{r} 81 \\ 12 \overline{)987} \\ \underline{96} \\ 017 \\ \underline{012} \\ 005 \end{array}$$

Ans: 81 remainder = 5

$$\therefore \frac{987}{12} = 81 \frac{5}{12}$$

(iii) $987 \div 111$

$$\begin{array}{r} 8 \\ 111 \overline{)987} \\ \underline{888} \\ 099 \end{array}$$

Ans: 8 remainder = 99

$$\therefore \frac{987}{111} = 8 \frac{99}{111}$$

11.2.2 Exercises: Following the method given above, do:

a. $\frac{8}{8}$ b. $\frac{9}{8}$ c. $\frac{88}{8}$ d. $\frac{889}{8}$ e. $\frac{98988}{8}$

f. $\frac{13}{12}$ g. $\frac{132}{12}$ h. $\frac{1335}{12}$ i. $\frac{12346}{12}$ j. $\frac{1000}{111}$

11.2.3 Example:

$$\frac{88}{24} = \frac{88}{8 \times 3} = \frac{88}{8} \times \frac{1}{3} = 11 \times \frac{1}{3} = \frac{11}{3} = 3 \frac{2}{3}$$

Do:

a. $\frac{889}{32}$ b. $\frac{98988}{48}$ c. $\frac{52}{24}$ d. $\frac{396}{36}$ e. $\frac{3000}{333}$ f. $\frac{3330}{999}$

11.3 Mixed fraction conversion

11.3.1

(a) $\frac{1}{2} + \frac{1}{2} = 1$ $1 \frac{1}{2} = 1 \frac{1}{2}$ Write this as

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2} \quad \therefore \quad \frac{3}{2} = 1 \frac{1}{2}$$

Try $1 \frac{1}{2} = \frac{2 \times 1}{2} + \frac{1}{2} = \frac{3}{2}$

This way we are able to convert mixed fraction (i.e. integer + fraction) into one single fraction.

11.3.2 Convert the mixed fractions into simple fractions.

$$2 \frac{1}{2}, 1 \frac{1}{4}, 1 \frac{1}{2}, 1 \frac{3}{4}, 2 \frac{3}{4}, 3 \frac{1}{2}, 4 \frac{3}{4}, 5 \frac{1}{2}, 9 \frac{1}{9}$$

Method:

$$2 \frac{1}{2} = 2 + \frac{1}{2} = \frac{(2 \times 2) + 1}{2} = \frac{4 + 1}{2} = \frac{5}{2}$$

$$1 \frac{1}{4} = 1 + \frac{1}{4} = \frac{(1 \times 4) + 1}{4} = \frac{4 + 1}{4} = \frac{5}{4}$$

$$4 \frac{3}{4} = 4 + \frac{3}{4} = \frac{(4 \times 4) + 3}{4} = \frac{16 + 3}{4} = \frac{19}{4}$$

$$9 \frac{1}{9} = \frac{(9 \times 9) + 1}{9} = \frac{81 + 1}{9} = \frac{82}{9}$$

$$11 \frac{1}{9} = \frac{(11 \times 9) + 1}{9} = \frac{99 + 1}{9} = \frac{100}{9}$$

11.3.3 Convert (i) $123 \frac{3}{8}$ (ii) $81 \frac{5}{12}$ (iii) $8 \frac{99}{111}$

Clue: Go to Para 14.2

$$\begin{array}{r} 3 \overline{) 81} \\ \underline{27} \end{array} \quad \text{This means } \frac{81}{3} = 27 \quad \text{Also } 3 \times 27 = 81$$

$$\begin{array}{r} 3 \overline{) 83} \\ \underline{27} \end{array} \quad (\text{remainder } 2) \text{ This means } 83 - 2 = 81 \quad \frac{81}{3} = 27 \quad \& \quad 3 \times 27 = 81$$

11.4 Equivalent fractions:

Fraction is the same as division into equal parts; otherwise called sharing or making equal heaps (Gudde).

11.5.1 If 2 chocolates are to be shared by 2 children how many will each child get?

Ans: $\frac{2}{2} = 1$

If there are 4 chocolates and 4 children? Ans: $\frac{4}{4} = 1$

If there are 52 items and 52 receivers Ans: $\frac{52}{52} = 1$

We get the same answer (viz 1) $\therefore \frac{2}{2} = \frac{4}{4} = \frac{52}{52} = 1$

11.5.2 If I have Re.1 and give to 2 persons, each gets?

Ans: Total = Re.1 Persons = 2 Each = Re $\frac{1}{2} = \frac{1}{2}$ rupee or 50 paise

If I have Rs. 5 and give to 10 persons? Ans: Rs. $\frac{5}{10} = \text{Re. } \frac{1}{2}$ or 50 paise

11.5.3 The above concept is sometimes called SIMPLIFYING a fraction. If 100 items are shared by 200 persons each = $\frac{100}{200}$. This can also be called $\frac{1}{2}$. Thus $\frac{100}{200} = \frac{1}{2}$ (simplified).

This is done by dividing both the numerators and the denominator by the same number.

Example: Simplify $\frac{8}{24}$

Ans: Divide numerator by 8; we get $\frac{8}{8} = 1$

Divide denominator by 8; we get $\frac{24}{8} = 3$

$\therefore \frac{8}{24} = \frac{1}{3}$

This can also be written as $\frac{8}{24} = \frac{8 \times 1}{8 \times 3} = \frac{1}{3}$

11.5.4 Exercises:

Do as shown above: Simplify

a. $\frac{2}{6}$ b. $\frac{2}{16}$ c. $\frac{4}{64}$ d. $\frac{42}{98}$ e. $\frac{999}{81}$ f. $\frac{72}{999}$

g. $\frac{18}{81}$ h. $\frac{18}{72}$ i. $\frac{72}{18}$

11.5.5 We saw how to simplify by DIVIDING BOTH TOP AND BOTTOM by the same number. Some times MULTIPLYING with help.

Example 1: Simplify $\frac{1234}{5}$. One can divide by long method. Instead, do as follows:

$$\frac{1234}{5} = \frac{1234 \times 2}{5 \times 2} = \frac{2468}{10}$$

$$= \frac{2460}{10} + \frac{8}{10} = 246 \frac{8}{10}$$

(Here it is easy for us to divide by 10)

Example 2: $\frac{12345}{125}$ ($125 \times 8 = 1000$ is known)

$$\begin{aligned}\therefore \frac{12345}{125} &= \frac{12345 \times 8}{125 \times 8} = \frac{88760}{1000} = \frac{88000 + 760}{1000} \\ &= 88 + \frac{760}{1000} \quad \text{Now Simplify } \frac{76}{100} \text{ only}\end{aligned}$$

Do:

a. $\frac{421}{5}$ b. $\frac{421}{25}$ c. $\frac{210}{25}$ d. $\frac{48}{50}$ e. $\frac{5250}{125}$

11.6 Some (real-life) examples:

- 11.6.1 Imagine this real life situation. You are giving 1 pencil per person (student). You have 10 students and 10 pencils (i.e. $\frac{10}{10} = 1$). Suddenly 5 more join. What will you do? 5 extra persons came; so you get 5 extra pencils.

$$\frac{10}{10} = 1 \quad \frac{?}{10+5} = 1 \quad \text{Answer is 15.}$$

$$\text{Extra needed} = 15 - 10 = 5$$

This looks like $\frac{10+5}{10+5} = \frac{10}{10}$ (i.e., adding to top & bottom is OK)

- 11.6.2 In 11.6.1 above, let us say we are giving 2 pieces (notebooks, laddus or ruppes) each person. We have 20 items and ten persons. Each gets 2 [i.e., $\frac{20}{10} = 2$]. Suddenly 10 more join. What will you do? You know the answer: you will get 20 more items.

Now let us try : $\frac{20}{10}$ was the first.

Extra came in $\frac{?}{10+10}$

If you make $(20 + 10)$ it becomes $\frac{20+10}{10+10} = \frac{30}{20}$. This is wrong.

It should be done like this: Each should get 2. Originally 10 persons, then we needed 20. We had 20. Now, 10 extra persons came. Now we have $(10+10) =$ total 20 persons. Each gets 2. So we need $(20 \times 2) = 40$ items. We had with us 20. \therefore we need 20 more.

- 11.6.3 In 11.6.2 above, only one item per person was being given i.e., $\frac{10}{10} = 1$. Here both the top number (= numerator) and the bottom number (= denominator) were the same. So, just adding was OK.

In 11.6.2, we were giving more than one per person i.e., numerator was not equal to denominator (i.e., it was a true fraction). In such cases adding will not be OK.

- 11.6.4 Imagine you cooked 10 idles for 10 persons. 5 did not come. How many will be left $\frac{10}{10} = 1$ (Planned) Now $\frac{?}{10-5}$. Answer 5 is OK.

Suppose you planned to serve 3 idles per person and expected 10 persons 5 did not come. How many will be wasted?

Answer is NOT 5. It will be 15.

$$\frac{30}{10} = 3(\text{OK}) \quad \frac{30-5}{10-5} \neq 3(\text{Not OK})$$

11.6.5 From the above 4 paragraphs (11.6.1 to 11.6.4), we learn that: ADDING OR SUBTRACTING to a fraction (numerator and denominator) gives wrong results.

11.7.1 Mathematically, the situations given above can be given as: $\frac{10}{10}$ means 10 items shared by 10 receivers. If receivers number increases by 5, $\frac{10}{10}$ becomes $\frac{?}{10+5}$ If it is = 15, adding 5 is OK.

Similarity 11.6.4 can be written as 10 shared by 10. If 2 are less, receivers number becomes 8.

$$\frac{10}{10} \text{ becomes } \frac{?}{10-2} = \frac{?}{8} \text{ If is } = 8$$

(reducing or subtracting 2 is OK)

All this because $\frac{10}{10}$ i.e., the original fraction was unity (=1, one). It is a unique case

(sometimes called TRIVIAL). For any other fraction, whether >1 or < 1, these methods do not work.

11.7.2 Caution: Adding the same number to both the numerator and the denominator is not OK. Doing that changes the value of the fraction. Similarly subtraction is also wrong.

11.7.3 To clarify still more:

$$\text{We know } \frac{3}{1} = 3 \text{ and } \frac{30}{10} = 3$$

$$3 \times 10 = 30 \text{ bottom also } 1 \times 10 = 10$$

$$\text{But } 3 + 27 = 30 \text{ bottom } 1 + 27 = 28 \quad \frac{30}{28} \neq 3$$

11.7.4 We can DIVIDE both denominator and numerator by the same number. It is OK.

$$\frac{30}{10} = 3 \quad \begin{array}{l} \text{Divide 30 by 10 Ans.: 3} \\ \text{Divide 10 by 10 Ans.: 1} \end{array} \quad \text{Now } \frac{3}{1} = 3$$

$$\frac{30}{10} = \frac{6}{2} = 3 \quad \text{OK} \quad \begin{array}{l} \text{Divide 30 by 5 Ans.: 6} \\ \text{Divide 10 by 5 Ans.: 2} \end{array}$$

11.7.5 Rule:

In fractions, multiplying or dividing both the numerator and denominator is OK.